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Research Article

A Fourier-Polynomial Time Series Regression Approach to Inflation Forecasting in Nigeria

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Abstract

Abstract: The inflation rate in Nigeria has persistence, nonlinear exchange-rate effects, and large seasonal variations, making the forecasting of Nigeria's inflation rate hard using standard linear time series models. Models of this type usually capture only one of the features and produce unstable forecasts and, importantly for developing economies, the impact of the exchange rate on inflation rate. A Fourier-Polynomial Time Series Regression (FP-TSR) model is proposed, which incorporates autoregressive dynamics, polynomial nonlinearity, interaction effects and Fourier components into a single framework. We analysed the monthly inflation and exchange rate data from the World Bank January 2004 to December 2024. Model parameters were estimated using ordinary least squares. Model fit was evaluated using the coefficient of determination (R^2) and adjusted R^2, while forecasting performance was assessed using root mean squared error (RMSE), mean absolute error (MAE), and mean absolute percentage error (MAPE). The results reveal strong inflation persistence and statistically significant nonlinear exchange-rate effects. The FP-TSR model achieves an R^2 of 0.933. In forecasting performance, it reduces RMSE, MAE, and MAPE by 1.41%, 3.06%, and 3.40%, respectively, relative to the lagged regression model. These findings demonstrate consistent improvement in both explanatory power and predictive accuracy compared to polynomial, Fourier-only, and lagged regression models. The proposed model significantly improves inflation forecasting performance and provides a robust, policy-relevant tool for short-term inflation forecasting in Nigeria and offers a flexible structure for modelling macroeconomic time series exhibiting similar variation.

Keywords: Fourier-Polynomial, Exchange rate, Fourier, Nonlinear, Time Series, Inflation rate

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1 INTRODUCTION

According to IMF [1], inflation forecasts remain very important to help stabilise the economy as a whole, especially for emerging countries, where price changes have impact on welfare, exchange rate stability, and monetary policy credibility. Historically on the Nigeria scene, inflation dynamics have revealed exchange-rate pass-through, structural supply shocks, fiscal pressures, and seasonality. Short-term forecasts become increasingly unreliable due to these features generating persistent nonlinear cyclical behaviours. Modeling is academically meaningful, but it is also a policy necessity [2].

Recent evidence is that inflation in emerging economies displays a strong degree of inertia. AR model often are used to capture this feature The Box-Jenkins methodology has been instrumental in modern time series analytics since 1970. The autoregressive representation formalised in the ARIMA framework allows serial dependence to be explicitly modelled. This method has a wide range of application in macroeconomic forecasting. Despite that, strictly linear AR structures assume constant marginal effects and symmetric adjustments. As a result, the assumption may be unsatisfactory in those economies where exchange-rate shocks are nonlinearly transmitted to domestic prices.

Evidence abounds that macroeconomic linkages do not run linearly. Polynomial regression offers a straightforward and flexible method for estimating nonlinear responses without imposing any rigid functional forms. Polynomial regression has been shown to model curvature in economic relationships with interpretability [3]. In modeling inflation, some researchers suggest nonlinear exchange rate pass-through due to threshold effects, market rigidities or policy asymmetries [4]. Polynomial regression may model such curvature, but it does not inherently handle time dependence.

The other thing that matters for the inflation series is how it varies over time and during the seasons. Traditional seasonal dummy variables often make changes that happen altogether rather than gradually. Methods employing Fourier series put to use a more sophisticated option of using sine and cosine functions to represent a periodic phenomenon. Fourier terms have been used in regression and time-series modelling since the classical harmonic analysis period.

Today, they are frequently employed for smooth cyclic processes [5]. Fourier based specifications have more recently been employed to capture structural shifts and gradual regime changes in macroeconomic data [6, 7]. In panel and time-series cases, Fourier functions have shown efficiency in approximating unknown structural breaks and cyclical components without overparameterisation [8].

In addition to modelling separate effects, interaction regression provides a framework for one variable working off the others. The use of interaction terms is an important development in contemporary regression research facilitating different marginal effects in different regimes or cycles [9]. Considering inflation, the exchange-rate effect may experience seasonality shifts. For example, when there is high demand for a certain price, the price is likely to be more responsive to depreciation. As a result, the structural realism that can be incorporated in interaction terms would allow a policy-relevant interpretation.

Although each of these modelling strategies – autoregressive dynamics, polynomial regression, Fourier representations and interaction effects – has been examined extensively in isolation, fewer studies merge them into the same inflation forecasting framework. Traditional ARIMA models gain persistence but lose nonlinear covariate response. Polynomial regression accounts for curvature, but not serial correlation. The interaction effects capture conditional effects, however, they do not impose a dynamic structure per se. The fact that no framework accommodates these features jointly motivates this paper.

This study introduces a Fourier-polynomial time series regression (FP-TSR) model that integrates:

1. Autorregressive specifications to capture inflation persistence.;
2. Polynomial exchange rate terms to model nonlinear pass-through;
3. Fourier sine and cosine functions for seasonal-cyclical variations that are smooth;
4. The interaction between the exchange rate and fourier components allows for seasonally varying marginal effects.

The FP-TSR generates an increasing flexibility while holding on to easy interpretability by combining the above elements within one OLS specification. The shape makes it possible to directly interpret persistence, curvature and cyclic modulation effects economically, unlike black box machine learning type approaches. The model is estimated using Nigerian data from January 2004 to December 2024. The performance of the model is evaluated against alternatives of lagged regression, polynomial only and Fourier only based on degree of explanatory power and forecasting performance.

The contribution of this study is therefore methodological and empirical. Methodologically, it demonstrates how classical econometric tools can be systematically integrated to address multiple empirical features of macroeconomic time series. Empirically, it provides evidence that jointly modelling persistence, nonlinearity, seasonality, and conditional effects improves inflation forecasting accuracy in Nigeria.

2 MATERIALS AND METHODS

2.1 Fourier-Polynomial Time Series Regression (FP-TSR) Model

The Fourier-Polynomial Time Series Regression (FP-TSR) model is defined below:

$$y_t = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-p} + \beta_1 \cos\left(\frac{2\pi t}{12}\right) + \beta_2 \sin\left(\frac{2\pi t}{12}\right) + \lambda_1 x_t + \lambda_2 x_t^2 + \theta_1 x_t \cos\left(\frac{2\pi t}{12}\right) + \theta_2 x_t \sin\left(\frac{2\pi t}{12}\right) + \varepsilon_t \tag{1}$$

The autoregressive component is

$$\alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-p} \tag{2}$$

The autoregressive component effectively captures inflation persistence, meaning that current inflation is influenced by its own past realisations.

Table 1: List of Parameters

Variables	Meaning
y_t	Monthly inflation rate at time t
β_0	Intercept term
β_1, β_2	Fourier coefficients
α_i	Autoregressive coefficient
y_{t-i}	Lagged inflation values
x_t	Exchange rate (NGN/USD)
λ_1 and λ_2	Polynomial coefficients
θ_1 and θ_2	Interaction coefficients
p	Order of autoregressive model
$\cos\left(\frac{2\pi t}{12}\right), \sin\left(\frac{2\pi t}{12}\right)$	Fourier functions
ε_t	Error term

The Fourier terms are

$$\beta_1 \cos\left(\frac{2\pi t}{12}\right) + \beta_2 \sin\left(\frac{2\pi t}{12}\right) \tag{3}$$

This models the seasonal-cyclical fluctuations in inflation without using any dummy variables and it also allows smooth oscillation over a 12-month cycle.

The polynomial terms are

$$\lambda_1 x_t + \lambda_2 x_t^2 \tag{4}$$

This is a quadratic polynomial in the exchange rate variable x_t. It simply represents nonlinear exchange-rate effects.

The interaction terms

$$\theta_1 x_t \cos\left(\frac{2\pi t}{12}\right) + \theta_2 x_t \sin\left(\frac{2\pi t}{12}\right) \tag{5}$$

These are seasonal interaction effects and they allow the marginal effect of x_t on inflation to vary across the seasonal cycle. The parameters of the model were estimated using the ordinary least squares method (OLS). The efficiency and performance of the FP-TSR model are examined using monthly data obtained from the World Bank for the period January 2004 to December 2024. Inflation is measured as the monthly percentage change in the Consumer Price Index (CPI), and the exchange rate is expressed in NGN/USD. Two autoregressive lags were selected to reflect observed inflation persistence while maintaining model parsimony. First-order and second-order polynomial terms were included to capture nonlinear effects without introducing excessive curvature or overfitting. The data were visualised with time plots to check if they exhibit seasonal and cyclical variations.

2.2 Model Diagnostics

The model was diagnosed to ensure the model's reliability and accuracy, so therefore, the model's stability, suitability, and residual performance will be evaluated using the approaches below.

2.2.1 Durbin–Watson Statistic

The existence of serial correlation in a model's residual is assessed by the Durbin-Watson statistic. It is given by

$$d = \frac{\sum_{t=2}^{T}(\varepsilon_t - \varepsilon_{t-1})^2}{\sum_{t=1}^{T}\varepsilon_t^2} \tag{6}$$

where T is the total number of observations.

2.2.2 Autocorrelation Function (ACF)

The autocorrelation function measures the linear relationship of a time series with its past values. It is defined as

$$r_k = \frac{\sum_{t=k+1}^{T}(y_t - \bar{y})(y_{t-k} - \bar{y})}{\sum_{t=1}^{T}(y_t - \bar{y})^2} \tag{7}$$

where r_k is the autocorrelation coefficient at lag k.

2.2.3 Coefficient of Determination (R^2)

The R^2 is the proportion of the total variation in the dependent variable that the proposed explanatory variables can explain. It is expressed as

$$R^2 = 1 - \frac{SSE}{SST} \tag{8}$$

where SSE is the sum of squared errors and SST is total sum of squares.

2.2.4 Adjusted Coefficient of Determination ($\bar{R}^2$)

The coefficient of determination or R-squared is a measure of how well the model explains the variations in the data being estimated. It is given by

$$\bar{R}^2 = 1 - \frac{(1 - R^2)(T - 1)}{T - k - 1} \tag{9}$$

where k is the number of explanatory variables and T is the sample size.

2.2.5 Forecast Evaluation Metrics

The study uses Mean Absolute Error (MAE), Root Mean Squared Error (RMSE) and Mean Absolute Percentage Error (MAPE) to assess forecast accuracy, defined respectively as

$$MAE = \frac{1}{T}\sum_{t=1}^{T}(\hat{y}_t - y_t)^2 \tag{10}$$

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T}(y_t - \hat{y}_t)^2} \tag{11}$$

$$MAPE = \frac{100}{T}\sum_{t=1}^{T}\left|\frac{(\hat{y}_t - y_t)}{\hat{y}_t}\right| \tag{12}$$

The performance of the forecasts is evaluated using out-of-sample approach that is training-testing split approach.

3 Results and Discussion

A visual inspection of the Nigerian monthly inflation rate and exchange rate plot in Figures 1 and 2 from January 2004 to December 2024, sourced from the World Bank Organisation, signified a non-stationary series. Figure 1 shows the non-stationary and cyclical behaviour of Nigeria's inflation rate, as indicated by the continuous rise and fall of this economic data. Cycles were completed on a monthly basis, and there is a significant continuous rise in the inflation rate from 2004.

Figure 2 represents corresponding movements in the exchange rate over the study period. That is, the exchange rate was slightly fluctuating and below two hundred naira per dollar.

Table 2 revealed that autocorrelation and partial autocorrelation coefficients for the shown delays are quite near to zero and well within the confidence intervals. There are no obvious spikes to show systematic serial reliance. Thus, there is no significant correlation among the residuals.

Table 2: Autocorrelation and Partial Autocorrelation Results

Lag	ACF	PACF	AC	PAC	Q-Stat	Prob
1			0.017	0.017	0.0706	0.807
2			-0.043	-0.043	0.4807	0.488
3			-0.050	-0.050	1.2384	0.266
4			0.041	0.042	1.6797	0.432
5			0.105	0.100	4.5508	0.208
6			0.058	0.056	5.4257	0.246
7			0.024	0.036	5.5795	0.349
8			0.052	0.066	6.2780	0.393
9			-0.074	-0.080	7.8448	0.346
10			0.003	-0.001	7.8469	0.449
11			-0.042	-0.050	8.2983	0.504
12			0.002	-0.020	8.2993	0.600
13			-0.001	-0.010	8.3034	0.686
14			0.032	0.037	8.5718	0.739
15			0.008	0.017	8.5893	0.803

In this model, Nigerian inflation rate is the response factor, while the exchange rate is the predictor series. The fitted FP-TSR model using the ordinary least squares estimation technique is

$$y_t = 0.5208 + 0.7393 y_{t-1} + 0.7926 \cos\left(\frac{2\pi t}{12}\right) - 0.1810 \sin\left(\frac{2\pi t}{12}\right) + 0.01507 x_t - 4.71 \times 10^{-06} x_t^2$$
$$- 0.0024 x_t \cos\left(\frac{2\pi t}{12}\right) + 0.0045 x_t \sin\left(\frac{2\pi t}{12}\right) \tag{13}$$

with $R^2 = 0.933011$, Adjusted $R^2 = 0.930509$ and Durbin Watson= 1.96855.
The coefficient of determination reveals that 93.3% of the variation in the inflation rate is effectively accounted for by the model, while the adjusted coefficient of determination suggests that the model is a good fit at 93.1%. The Durbin–Watson statistic suggests the absence of autocorrelation, as it closely approximates a standard value of 2.

In Table 3, the coefficient on lagged inflation ($y_{t-1} = 0.8295$, $p < 0.001$) is positive and highly statistically significant, but the second lag of inflation ($y_{t-2} = -0.0474$, $p = 0.427$) is statistically insignificant, this implies that the inflation rate is heavily dependent on its immediate past value. The exchange rate exhibits a statistically significant nonlinear relationship with inflation. The linear term ($x_t = -0.1198$, $p = 0.017$) is negative and significant, while the squared term ($x_t^2 = 0.0005$, $p = 0.015$) is positive and significant. the sine component ($\sin(2\pi t/12) = 1.3895$, $p = 0.196$) is not statistically significant, the cosine component ($\cos(2\pi t/12) = -2.5907$, $p = 0.025$) is significant at the 5% level. This suggests that inflation follows a seasonal pattern that is primarily phase-shifted rather than symmetric. The interaction terms between the exchange rate and the seasonal components are statistically insignificant.

Table 3: Coefficients and P-values of the fitted FP-TSR model

Variable	Coefficient	p-value
Constant	6.981	0.001
x_t	-0.1198	0.017
x_t^2	0.0005	0.015
$\sin\left(\frac{2\pi t}{12}\right)$	1.3895	0.196
$\cos\left(\frac{2\pi t}{12}\right)$	-2.5907	0.025
$x_t \sin\left(\frac{2\pi t}{12}\right)$	-0.0048	0.297
$x_t \cos\left(\frac{2\pi t}{12}\right)$	0.0003	0.824
y_{t-1}	0.8295	0.000
y_{t-2}	-0.0474	0.427

Figure 3 is the time plot of actual versus fitted inflation rate values. Both the trend and seasonal fluctuation of the series are effectively captured in the fitted series. The time plot also revealed that the actual and the fitted series are closely aligned, indicating that the model is adequate and supports the high overall model performance reported by the coefficient of determination and adjusted coefficient of determination. Minor deviations during sharp inflationary spikes suggest the presence of structural shocks.
When compared to the other models that were used in this study, Table 4 shows that the FP-TSR model coefficient and adjusted coefficient of determination had the greatest values. This suggests

that the combination of lagged values, polynomial terms, and Fourier components provides a more credible explanation of the data since the FP-TSR model better describes the change of the inflation rate and model complexity. Also the FP-TSR model improves explanatory power by 40.9% in coefficient of determination and 41.1% in adjusted coefficient of determination.

Percentage improvement of the next best (PINB) is simply equal to

$$\frac{\text{the model - the next best model}}{\text{the next best model}} \times 100 \tag{14}$$

Table 4: Values of Coefficient and Adjusted Coefficient of determination for FP-TSR Model

Model(s)	R^2	$\bar{R}^2$	PINB (R^2)	PINB ($\bar{R}^2$)
FP-TSR	0.933011	0.930509	40.9%	41.1%
Polynomial regression	0.662199	0.659474		
Lagged regression	0.653311	0.650504		
Fourier regression	0.039799	0.032056		

The FP-TSR model forecast evaluation measures were compared with existing models used in the study. Table 5 revealed that the FP-TSR model has the least value, indicating that it is a better model to forecast the inflation rate. The FP-TSR model records an RMSE of 37.20, representing a reduction of approximately 1.4% compared to the lagged regression model and over 38% compared to polynomial and Fourier regression models, indicating a measurable improvement in forecasting accuracy. The relatively high MAPE values reflect the sensitivity of percentage-based error measures to periods of low or highly volatile inflation, a known limitation when evaluating inflation forecasts in developing economies. In forecast evaluation for the FP-TSR model, the percentage improvement over the next best model reduces in RMSE, MAE, and MAPE by 1.41%, 3.06%, and 3.40%, respectively, compared to the lagged regression model. Therefore, the result shows that the integration of autoregressive, nonlinear, and seasonal components yields both statistically and practically meaningful improvements.

Table 5: Forecast Evaluation for FP-TSR Model

Model(s)	RMSE	MAE	MAPE	PINB (RMSE)	PINB (MAE)	PINB (MAPE)
FP-TSR	37.2	28.2	257.6772	1.41%	3.06%	3.40%
Lagged Regression	37.73174	29.09151	266.7330			
Polynomial Regression	60.00885	49.14192	457.3568			
Fourier Regression	62.74349	45.97751	390.2492			

4 Conclusion

This study developed an FP-TSR model to analyse and forecast Nigeria's inflation rate using monthly data from January 2004 through December 2024. The FP-TSR model overcomes significant drawbacks of traditional linear and single-structure methodologies by integrating linear terms, polynomial structures, interaction effects, and Fourier components into a unified specification. The findings indicate that the inflation rate in Nigeria exhibits significant persistence, nonlinearity, and seasonal-cyclical patterns. The Fourier terms accurately represent asymmetric seasonal and cyclical variations within a twelve-month cycle, underscoring the necessity of explicitly modelling periodic behaviour in an inflation series.

Model diagnosis indicates model adequacy and stability. The Durbin–Watson statistic, together with the autocorrelation findings, demonstrates that the model dynamic structure properly resolves any serial dependence since no residual autocorrelation coefficients are confirmed. The analysis reveals that the FP-TSR outscores polynomial, Fourier, and lagregression models in both explanatory power and forecasting accuracy. A high adjusted R-squared implies the superior performance is not due to over-parameterization, while low forecast error measures support its predictive accuracy.
. The results point out that in Nigeria, the inflation rate cannot be adequately accounted for by a single linear, lagged or seasonal model. A hybrid modelling approach that simultaneously incorporates persistence, nonlinearity, interaction effects, and seasonal cyclical variation provides a more accurate and informative representation of the inflation rate. Improved short-term inflation forecasts from the proposed model can support inflation targeting, monetary policy calibration, and early warning systems used by Nigerian policymakers. The univariate structure, monthly frequency, and assumption of a fixed seasonal cycle limit this study. Future research may incorporate additional macroeconomic predictors, higher-order Fourier terms, or cross-country applications.

Availability of Code

A mixture of statistical tools were used to conduct the empirical analysis. Model estimation and core computations were implemented in Python (version 3.x) using standard scientific libraries like NumPy, Pandas, Statsmodels, and Matplotlib. The model was estimated, diagnostic tests performed and graphical visualisation produced in Jupyter Notebook.

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Author Contributions

All authors had significant contributions in completing this manuscript.

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Conflicts of Interest

The authors declare no conflict of interest.

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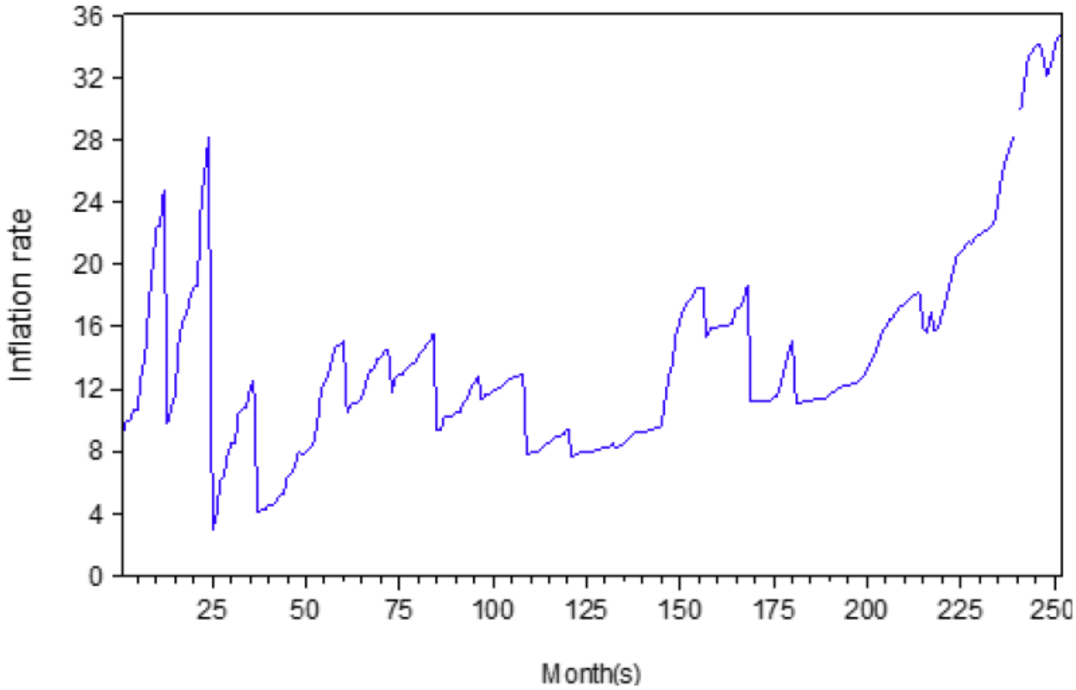

Figure 1: Time plot of Nigerian Inflation rate from January 2004 to December 2024

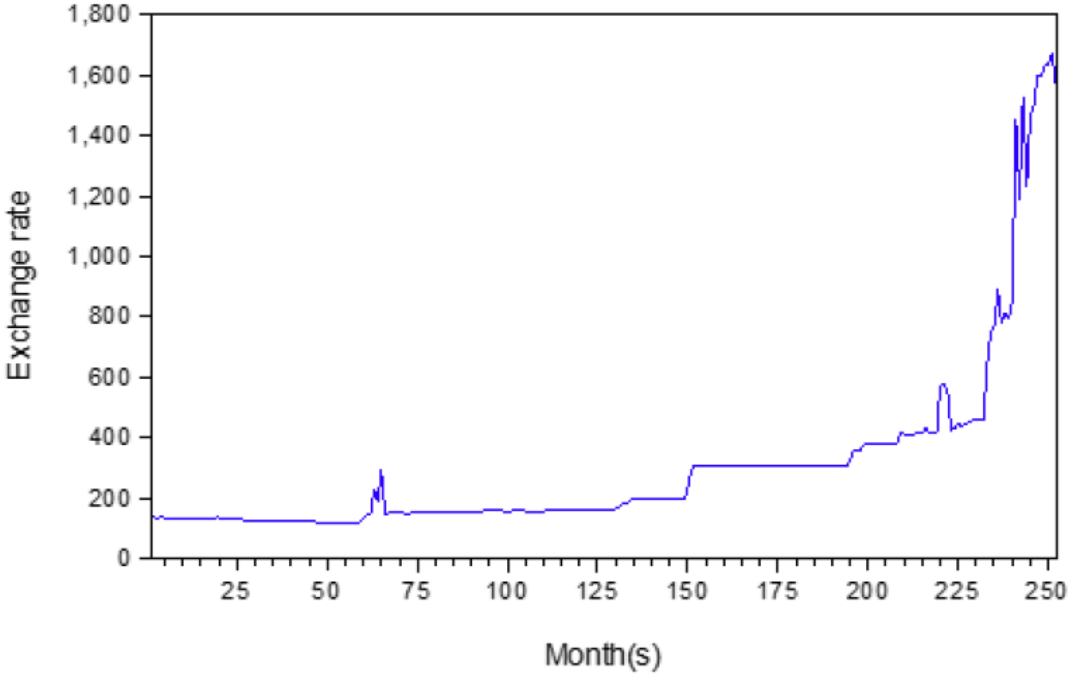

Figure 2: Time plot of Nigerian exchange rate from January 2004 to December 2024

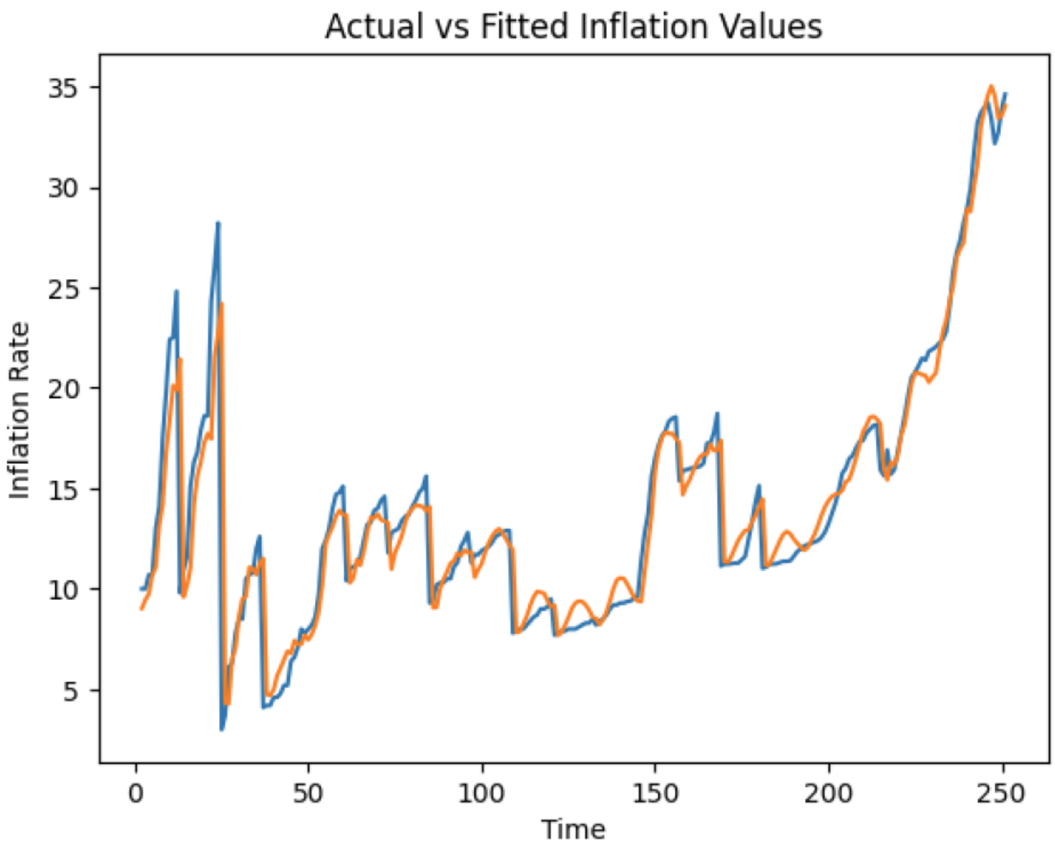

Figure 3: Time Plot of Actual versus Fitted Values of Inflation Rate