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Research Article

Time Series Fourier Regression Modeling of the Effect of Exchange Part on Nigerian Economic Growth

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Abstract

Abstract: Macroeconomic datasets have been examined using various techniques. Traditional econometric methods were unable to effectively identify cyclical and periodic patterns within the data, leading to less accurate forecasts. This study addresses this gap by applying a Time Series Fourier Regression (TSFR) model to evaluate the impact of exchange rate fluctuations on Nigeria's economic growth. Annual data on Gross Domestic Product (GDP) and official exchange rates from 1960 to 2023 were obtained from the Central Bank of Nigeria (CBN). The TSFR model was estimated using the Ordinary Least Squares (OLS) method and validated through the Durbin-Watson (DW) statistic, residual histograms, and Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) checks. Model performance was benchmarked against multiple regression, lagged regression and multiple Fourier regression models using Root Mean Square Error (RMSE), Mean Absolute Error (MAE), and Mean Absolute Percentage Error (MAPE). It was revealed that exchange rate fluctuations significantly influence GDP. The TSFR model explained over 80% of GDP variability, with an adjusted coefficient of determination (R-squared) of 78%. Diagnostic tests confirmed normally distributed residuals without serial correlation. Comparative analysis demonstrated that the TSFR model consistently outperformed alternative models in terms of explanatory power and forecast accuracy. The results gives reliable insights into the dynamics between exchange rates and economic growth. The study establishes that Fourier-based time series models provide a superior framework for analysing macroeconomic data, effectively capturing both secular and cyclical movements.

Keywords: Exchange Rate Gross Domestic Product, Fourier model, inflation, Time series, Forecast.

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1 Introduction

A key macroeconomic variable that influences a country's economic performance is its currency exchange rate [1, 2]. The exchange rate between two currencies determines a country's competitive standing in international trade. Within an open economy, where demand for foreign currency exceeds supply, fluctuations in the exchange rate can have a significant impact on economic activities and growth (Ewubare and [3, 4]). In the context of Nigeria operating a single-culture nation where the government receives more than 80% of its foreign income derived from crude oil exports on the global market, making fluctuations in oil prices a significant factor affecting the country's economy [5, 6]. On the other hand, the nation's economy relies on imports of technological advances, mineral resources, and various associated products from different countries. The cyclical nature related to fluctuations in the exchange rate have frequently been caused by the strain on foreign exchange resulting from inadequate foreign earnings to fulfil demands [7, 8, 9, 10]. This situation stated above has made Nigeria's economy suffer over time from the effects of ongoing fluctuations in exchange rates, unpredictability in macroeconomic factors, and generally weak economic expansion as compared with emerging economies with predictable exchange rate [10]. To gain insight into the significant impact of exchange rate fluctuations on Nigerian economic growth, various studies have employed methods such as time series econometrics and different time series models over the years[11, 12]. However, these models, which are frequently used to assess the extent of exchange rate fluctuations and their tangible effects on economic expansion, may not be sufficiently adequate since the temporal and periodic structures seen in the datasets are not often catered for before analysis. However, a time series Fourier regression model which combines autoregressive modelling with Fourier analysis has been developed to handle the temporal and periodic structures present in Nigerian exchange rate datasets. The model has four stages: identification, estimation, diagnostics, and forecasting.

2 Literature Review

In the Nigerian context, Akpan (2009) found that fluctuations in exchange rates adversely affect economic growth, primarily through its effect on trade and investment [2]. Furthermore, Adeniran et al. (2014) provided evidence that exchange rate fluctuations adversely affect the economic outcomes of Nigeria by increasing the cost of imports and creating uncertainty for investors [3]. The findings by Ufoeze et al. (2017), who investigated how variations in exchange rates affect economic growth in Nigeria using annual data spanning from 1986 to 2012, imply that there is not much of a significant relationship between Nigeria's economic growth and currency rates [13].

Fourier regression has been applied in various economic contexts. For instance, Franses (1996) used Fourier terms to model seasonal patterns in macroeconomic time series [14]. Similarly, Fokianos and Tjøstheim (2010) demonstrated the utility of Fourier series in capturing periodic structures in economic data, thereby improving forecast accuracy and enhancing the understanding of underlying economic processes [15].

3 Methodology

The proposed time series Fourier regression model in this study is specified as

$$y_t = C_0 + C_1 X_t + \sum_{i=1}^{k} \beta_i X_{t-i} + \sum_{i=1}^{k} \left(\alpha_i \cos(\omega X_t) + \delta_i \sin(\omega X_t)\right) + \mu_t, \quad t = 1, 2, \ldots, P \tag{1}$$

where y_t is the dependent variable, X_t and X_{t-i} are the independent variables, $\cos(\cdot)$ and $\sin(\cdot)$ are the Fourier components used to capture cyclical patterns, $\omega = \frac{2\pi}{n}$ is the periodic frequency, C_0, C_1, β_i, α_i, and δ_i are the model coefficients, and μ_t is the residual term.

3.1 Model Building in Time Series Fourier Regression Model

The four basic steps in analysing the Time Series Fourier Regression model are carried out as follows.

3.2 Model Identification for Time Series Fourier Regression Model

Data exploration was conducted on the dataset by utilising time plots to visualise the exchange rate over a specific period. This approach helped in identifying the types of variations and patterns present in the data, such as trends, seasonal fluctuations, or irregular movements. By examining these visual representations, valuable insights were gained into the behaviour of the exchange rate, which informed subsequent modelling and analytical decisions.

3.3 Model Estimation for Time Series Fourier Regression Model

The coefficients of the model were determined using the Least Squares Estimation (LSE) method. This approach involves finding the set of coefficient values that minimise the sum of the squared differences between the observed values and the predicted values generated by the model, known as the residuals. By minimising the residual term, the least squares method ensures that the overall prediction error is as small as possible, thereby leading to the most accurate fit of the model to the data.

This technique provides a systematic and efficient way to estimate the model parameters, thereby enhancing the model's predictive accuracy and reliability in capturing the underlying relationships within the data.

$$\begin{pmatrix} C_0 \\ C_1 \\ B_1 \\ \alpha_1 \\ \delta_1 \end{pmatrix} = \begin{pmatrix} n & \sum X_t & \sum X_{t-1} & \sum \cos(\omega X_t) & \sum \sin(\omega X_t) \\ \sum X_t & \sum X_t^2 & \sum X_t X_{t-1} & \sum X_t \cos(\omega X_t) & \sum X_t \sin(\omega X_t) \\ \sum X_{t-1} & \sum X_t X_{t-1} & \sum X_{t-1}^2 & \sum X_{t-1} \cos(\omega X_t) & \sum X_{t-1} \sin(\omega X_t) \\ \sum \cos(\omega X_t) & \sum X_t \cos(\omega X_t) & \sum X_{t-1} \cos(\omega X_t) & \sum \cos^2(\omega X_t) & \sum \cos(\omega X_t) \sin(\omega X_t) \\ \sum \sin(\omega X_t) & \sum X_t \sin(\omega X_t) & \sum X_{t-1} \sin(\omega X_t) & \sum \cos(\omega X_t) \sin(\omega X_t) & \sum \sin^2(\omega X_t) \end{pmatrix}^{-1} \tag{2}$$

3.4 Model Validation

The validation of the models was primarily based on their Coefficient of Determination (R^2) and Adjusted Coefficient of Determination ($\bar{R}^2$) values. These statistical metrics measure the proportion of variance in the dependent variable that can be explained by the independent variables included in the model.

A higher R^2 indicates that the model explains a greater proportion of the variability in the data, while the Adjusted R^2 accounts for the number of predictors used, thereby providing a more accurate assessment of the model's explanatory power, especially when multiple variables are involved.

By comparing these values across different models, the relative performance of the models was evaluated to determine which specification best fit the data, thereby ensuring robustness and reliability in the predictive capabilities of the models.

3.4.1 Coefficient of Determination for TSFR Model

The Coefficient of Determination for the TSFR model is obtained as

$$R^2 = \frac{\hat{\boldsymbol{\beta}}'(X'X)\hat{\boldsymbol{\beta}} - n\bar{y}^2}{y'y - n\bar{y}^2} \tag{3}$$

3.4.2 Adjusted Coefficient of Determination for TSFR Model

The Adjusted Coefficient of Determination is given by

$$\bar{R}^2 = 1 - \frac{n-1}{n-k}(1 - R^2) \tag{4}$$

3.5 Model Diagnostic and Forecast Evaluation

3.5.1 Error Term Performance

If ε_t denotes the residual for the observation at time t, then the Durbin–Watson test statistic is given by

$$d = \frac{\sum_{t=2}^{T}(\varepsilon_t - \varepsilon_{t-1})^2}{\sum_{t=1}^{T}\varepsilon_t^2} \tag{5}$$

where T denotes the number of observations. For large sample sizes, the statistic can be linearly related to the residual autocorrelation in the time series. The Durbin–Watson statistic ranges from 0 to 4 and satisfies the relationship

$$d = 2(1 - r),$$

where r is the sample autocorrelation of the residuals. A value of d close to 2 indicates the absence of autocorrelation. Values less than 2 suggest positive autocorrelation, while values greater than 2 indicate negative autocorrelation.

After specifying a time series model and estimating its parameters, it is necessary to conduct diagnostic tests to determine whether the original specification is adequate. A careful examination of the estimated residuals was therefore carried out to assess whether they follow a white noise process. This was achieved by computing the sample Autocorrelation Function (ACF) and Partial Autocorrelation Function (PACF) of the residuals.

The sample autocorrelation of the residuals, denoted by $\hat{\Gamma}_i(\varepsilon)$, is approximately normally distributed about zero with finite variance. At significance level $\alpha = 0.05$, the residual autocorrelations should lie within two standard deviations of zero. If the model is correctly specified, $\hat{\Gamma}_i(\varepsilon)$ should behave like a white noise process.

3.5.2 Forecasting Evaluation

In achieving this, there are some measurements of the accuracy of forecast that will be applied, these are Root Mean Square Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE)

$$\text{RMSE} = \sqrt{\frac{1}{t+1}\sum_{t=1}^{p-1}(\hat{X}_t - X_t)^2} \tag{6}$$

$$\text{MAPE} = \left|\sum_{t=1}^{p-1}\frac{\hat{X}_t - X_t}{\hat{X}_t}\right| \tag{7}$$

$$\text{MAE} = \frac{1}{t+1}\sum_{t=1}^{p-1}|\hat{X}_t - X_t| \tag{8}$$

where $t = 1, 2, \ldots, p-1$. The exact value and predicted values for corresponding t values are denoted by $\hat{X}_t$ and X_t respectively. The smaller the values of RMSE, MAPE, and MAE, the better the forecasting performance of the model.

4 Results and Discussions

In order to determine the efficiency of the Time Series Fourier Regression (TSFR) model, a visual observation of the Nigeria exchange rate and Nigeria Gross Domestic Product datasets from 1960 to 2023 obtained from Central Bank of Nigeria (CBN) was analysed. As shown in Figure 1 and Figure 2, these economic series exhibited secular and cyclical variations, and this was indicated by continuous rises, falls and fluctuations. Nigerian exchange rate signify a steady risen trend but becomes cyclical with a sharp rise in 1998. The GDP also exhibits a continuous trend before a unique rise in 1980.

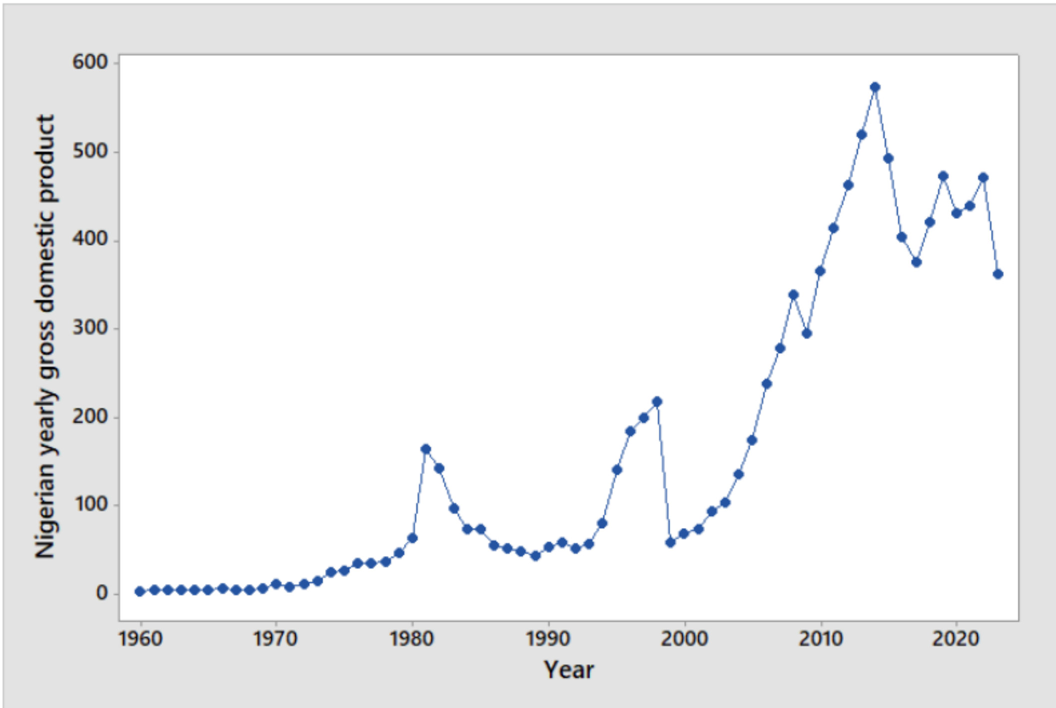

Figure 1: Time plot of Nigerian yearly gross domestic product from 1960 to 2023

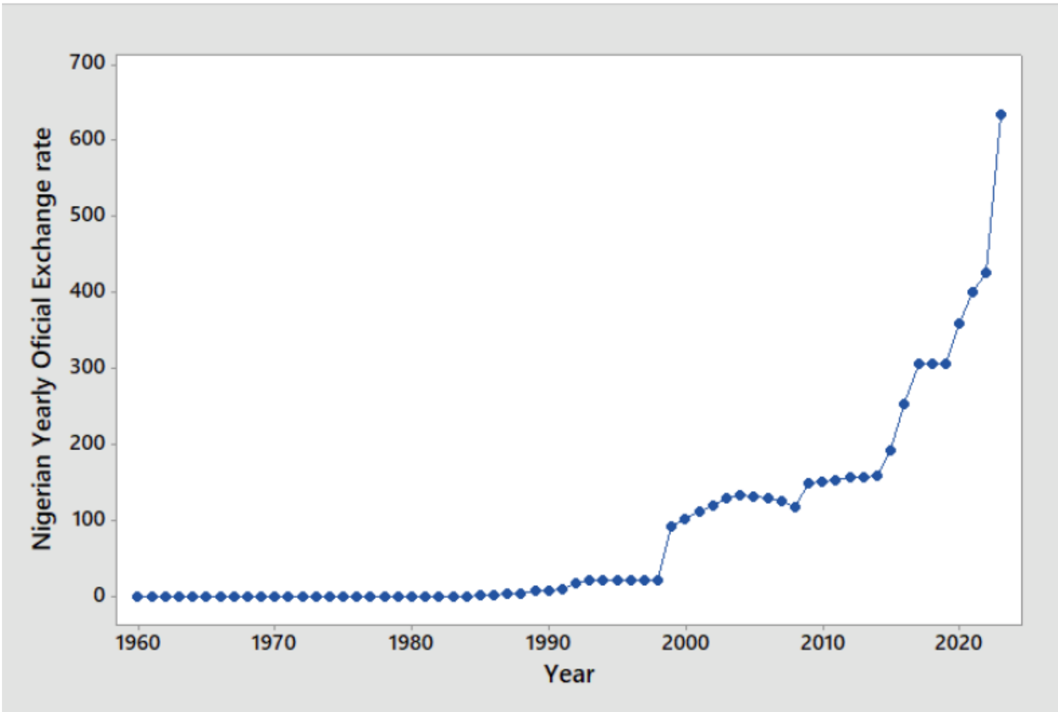

Figure 2: Time plot of Nigerian yearly official exchange rate from 1960 to 2023

The descriptive analysis of the Nigerian yearly gross domestic product and exchange rate given in Table 1. This signifies that the means are 160.30 and 86.0, standard deviation are 170.7 and 130.5, and the minimum are 0.50 and 4.20 and maximum are 633.83 and 574.20 respectively.

Table 1: Descriptive analysis of Nigerian yearly gross domestic product and exchange rate

Variable	Mean	Standard Deviation	Minimum	Maximum
Official exchange rate	86.000	130.50	0.50	633.83
Gross domestic product	160.30	170.70	4.20	574.20

4.1 Model Estimation and Validation

The proposed TSFR approach is utilised for modeling the effects of trend and cyclical movements in Nigerian exchange rate on Nigeria gross domestic product. In this model, GDP stands as the response factor, while exchange rate is taken as the predictor series. The TSFR model utilised in the study is detailed as

$$y_{gdp} = C_0 + C_1 X_{exg} + B_1 X_{(exg-1)} + \alpha_1 \cos\left(\frac{\pi}{6}\right) X_{exg} + \delta_1 \sin\left(\frac{\pi}{6}\right) X_{exg} + \mu_t \tag{9}$$

where the period is $\frac{2\pi}{12} = \frac{\pi}{6}$.

The fitted TSFR model from the Ordinary Least Square estimation methods is

$$y_{gdp} = 61.8 + 0.571 X_{exg} + 0.612 X_{(exg-1)} - 45.9 \cos\left(\frac{\pi}{6}\right) X_{exg} + 30.1 \sin\left(\frac{\pi}{6}\right) X_{exg} \tag{10}$$

where y_{gdp} is GDP (dependent variable) , X_{exg} is current period value of an exchange rate , and $X_{(exg-1)}$ is previous period value of an exchange rate.

For each one unit in current exchange rate, the model predict an increase of approximately 0.51 units in GDP before considering the trigonometric function. Also, a unit increase with lagged exchange rates leads to an increase in approximately 0.612 units in GDP.

The model suggests that X_{exg} has a strong negative impact on y_{gdp} while $X_{(exg-1)}$ has a positive but smaller influence on GDP. The combined coefficient after considering the trigonometric components indicate the net effect of X_{exg} is significantly negative.

$$R^2 = 0.80254, \quad \bar{R}^2 = 0.78240, \quad \text{Durbin Watson} = 1.739223$$

The value of the coefficient of determination (R^2) in the equation revealed that exchange rate accounted for variability within Nigerian yearly gross domestic product as much as 80%. While the adjusted coefficient of determination ($\bar{R}^2$) value demonstrated the TSFR approach also has high goodness of fit with a 78% level of predictive power. The value of the Durbin-Watson statistics displayed indicated that the residuals are not correlate serially.

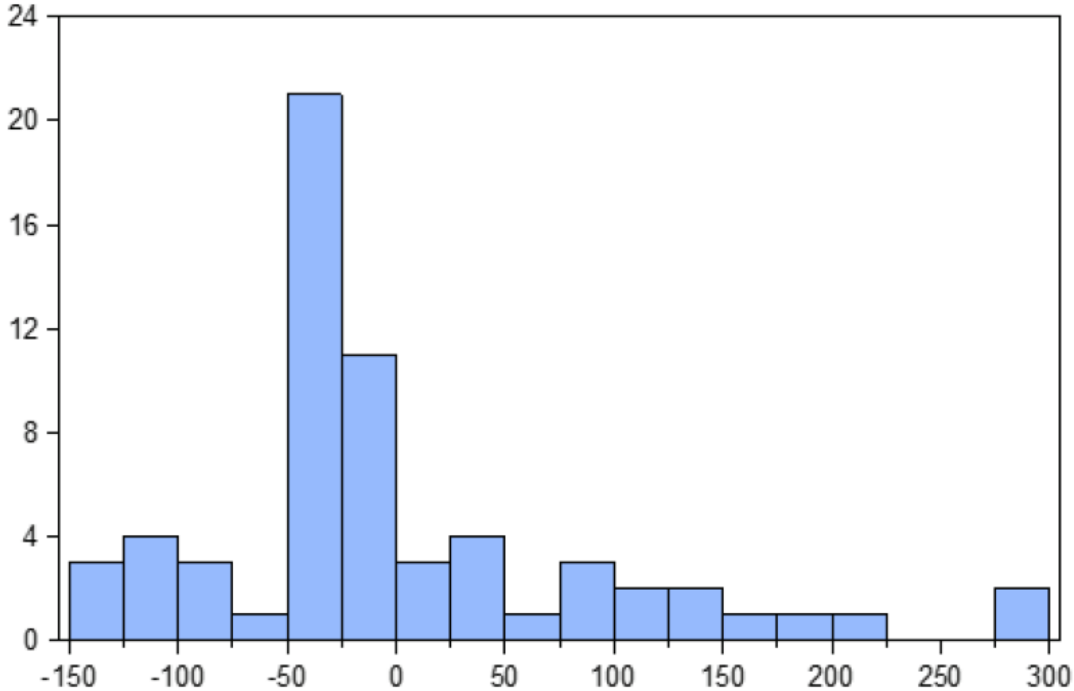

Figure 3: Time plot of Nigerian yearly official exchange rate from 1960 to 2023

This histogram of the residual for the fitted time series Fourier regression model in Figure **??** implied that the residuals are distributed normally since the histogram of residual exhibited a bell shape.

Table 2: Correlogram of Residuals

Lag	Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
1			0.789	0.789	41.150	0.000
2			0.523	-0.262	59.544	0.000
3			0.303	-0.034	65.828	0.000
4			0.171	0.047	67.850	0.000
5			0.047	-0.140	68.006	0.000
6			-0.082	-0.136	68.566	0.000
7			-0.220	-0.140	72.416	0.000
8			-0.340	-0.120	81.394	0.000
9			-0.350	0.109	90.905	0.000
10			-0.300	-0.030	98.233	0.000
11			-0.280	-0.120	104.48	0.000
12			-0.240	0.062	109.23	0.000
13			-0.170	0.031	111.63	0.000
14			-0.080	-0.020	112.21	0.000
15			0.040	0.142	112.35	0.000

Also, the ACF and PACF plots of residuals showed the residuals did not form any usual structure and all are significant at $\alpha = 0.05$.

4.2 Model Comparison based on Model Validation and Forecast Evaluation Metrics for Fourier Time Series Regression Model

The time series Fourier regression model performance was compared with multiple Fourier regression, multiple regression analysis and lagged time series regression models with regards to coefficient of determination (R^2) and Adjusted coefficient of determination. The TSFR model was compared with the current model based on model validation and forecast evaluation as shown in Table 3. It was revealed that TSFR model explain the variation to about 80% coefficient of determination and 78% adjusted coefficient of determination values. The result in Table 4 explain the forecast evaluation value, the minimum value of Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Root Mean Square Error (RMSE) for TSFR model makes it better than the existing model.

Table 3: Values of Coefficient and Adjusted Coefficient of Determination for Time Series Fourier Regression Model

Model(s)	Coefficient of Determination (R^2)	Adjusted Coefficient of Determination ($\bar{R}^2$)
Time Series Fourier Regression (TSFR)	0.802540	0.782400
Lagged Time Series Regression (LTSR)	0.696467	0.686349
Multiple Regression (MR)	0.605728	0.599369
Multiple Fourier Regression (MFR)	0.166705	0.139384

Table 4: Forecast Evaluation for Time Series Fourier Regression Model

Model(s)	RMSE	MAE	MAPE (%)
Time Series Fourier Regression (TSFR)	71.16169	58.33477	0.3564
Lagged Time Series Regression (LTSR)	92.9752	69.92279	0.6245
Multiple Regression (MR)	106.3474	77.76424	0.3906
Multiple Fourier Regression (MFR)	154.6067	121.56770	0.7515

5 Summary and Conclusion

5.1 Summary

The Nigerian yearly gross domestic product and exchange rate time plot from 1960 to 2023 revealed that the series were not stationary. Both series exhibited trend and cyclical movement, and this showed a continuous rise and fall over the period under consideration. The TSFR was utilised to model the relationship between Nigerian yearly gross domestic product and exchange rate that exhibited trend and cyclical variations. The model in question regarded gross domestic product as the response factor and exchange rate as predictor. Estimation was achieved using the OLS method and the Durbin–Watson statistic value was used to indicate that the residuals do not exhibit serial correlation.

The TSFR model obtained indicated that for the predictor changed by one unit, there should be a unit rise or fall in gross domestic product on yearly bases. The R^2 value indicated that up to 80% of variability within Nigerian gross domestic product could be explained by exchange rate. Furthermore, a high goodness of fit for the model was shown by the $\bar{R}^2$ value with a 70% level of predictive power.

To further validate the model, the histogram, ACF and PACF of residuals were utilised for indicating that the residuals showed no discernible fluctuations and were significant at $\alpha = 0.05$. Then the fitted Fourier time regression model is considered adequate for modelling and forecasting Nigerian yearly gross domestic product.

The derived Fourier Time Series Regression (FTSR) model effectiveness was contrasted with Fourier regression analysis, Multiple regression analysis and Lagged Time Series Regression models in terms of R^2, $\bar{R}^2$ values and forecast evaluation metrics. The value of the coefficient of determination for the proposed FTSR model revealed that the predictor explained the variations in Nigerian gross domestic product series better than all the other models estimated. As well, $\bar{R}^2$ signified that the time series regression model has better goodness of fit and predictive ability than all other models estimated. The results of the forecast evaluation metrics (MAE, RMSE and MAPE) further indicated that the proposed Time Series Fourier Regression model will have better forecast accuracy than all other fitted models since it has the lowest values of forecast evaluation metrics.

5.2 Conclusion

The model was derived utilising the joining of lagged time series regression and Fourier regression models. The time plot was primarily utilised to establish the movement exhibited by the time series datasets. The Ordinary Least Squares method was employed to estimate the models. The Durbin–Watson statistic, the residual histogram, and the ACF and PACF of residuals were used to diagnose the model.

The model was validated utilising coefficient of determination, adjusted coefficient of determination and forecast evaluation metrics (MAE, RMSE and MAPE). The efficiency of the derived model was ascertained to be better for modelling and forecasting Nigerian gross domestic product in respect to exchange rate movement based on the values of coefficient of determination, adjusted coefficient of determination and forecast evaluation metrics when compared with those of lagged time series, multiple and Fourier regression models.

In conclusion, the model revealed that if the exchange rate increases, GDP tends to decrease, possibly reflecting the negative impact of currency depreciation on the economy.

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